Name ______________________ Class ______________ Date __________

5-4

The Triangle Midsegment Theorem

Focus on Reasoning

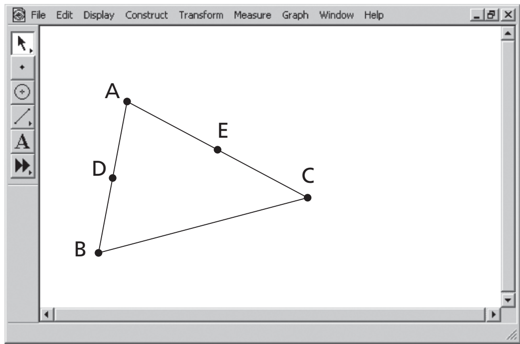
G-CO.3.10, G-GPE.2.4

Essential question: *What must be true about the segment that connects the midpoints of two sides of a triangle?*

A **midsegment** of a triangle is a line segment that connects the midpoints of two sides of the triangle.

1 Investigate midsegments.

A Use geometry software to draw a triangle.

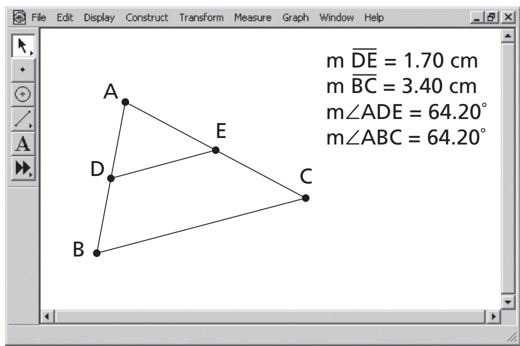


B Label the vertices *A, B,* and *C.*

C Select $\overline{AB}$ and construct its midpoint. Select $\overline{AC}$ and construct its midpoint. Label the midpoints *D* and *E.*

D Draw the midsegment, $\overline{DE}$.

E Measure the lengths of $\overline{DE}$ and $\overline{BC}$.



F Measure $\angle ADE$ and $\angle ABC$.

G Drag the vertices of $\triangle ABC$ to change its shape. As you do so, look for relationships in the measurements.

REFLECT

1a. How is the length of $\overline{DE}$ related to the length of $\overline{BC}$?

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1b. How is $m\angle ADE$ related to $m\angle ABC$? What does this tell you about $\overline{DE}$ and $\overline{BC}$? Explain.

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1c. Compare your results with those of other students. Then state a conjecture about a midsegment of a triangle.

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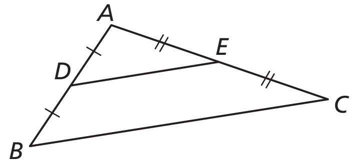
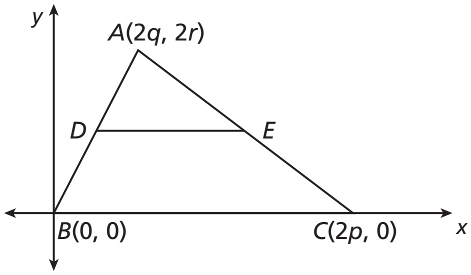
2 Prove the Midsegment Theorem.

A midsegment of a triangle is parallel to the third side of the triangle and is half as long as the third side.



Given: $\overline{DE}$ is a midsegment of $\triangle ABC$.
Prove: $\overline{DE} \parallel \overline{BC}$ and $DE = \frac{1}{2}BC$.

A Use a coordinate proof. Place $\triangle ABC$ on a coordinate plane so that one vertex is at the origin and one side lies on the x-axis, as shown. For convenience, assign vertex C the coordinates $(2p, 0)$ and assign vertex A the coordinates $(2q, 2r)$.



B Use the midpoint formula to find the coordinates of D and E. Complete the calculations.

$D\left(\frac{2q+0}{2}, \frac{2r+0}{2}\right) = D(q, r)$ $\quad\quad$ $E\left(\frac{\square + \square}{2}, \frac{\square + \square}{2}\right) = E\left(\square, \square\right)$

C To prove that $\overline{DE} \parallel \overline{BC}$, first find the slopes of $\overline{DE}$ and $\overline{BC}$.

Slope of $\overline{DE} = \frac{\square - \square}{\square - \square} = \square$

Slope of $\overline{BC} = \frac{\square - \square}{\square - \square} = \square$

What conclusion can you make based on the slopes? Why?

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D Show how to use the distance formula to prove that $DE = \frac{1}{2}BC$.

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REFLECT

2a. Explain why it is more convenient to assign the coordinates as $C(2p, 0)$ and $A(2q, 2r)$ rather than $C(p, 0)$ and $A(q, r)$.

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2b. Explain how the perimeter of $\triangle JKL$ compares to that of $\triangle MNP$.

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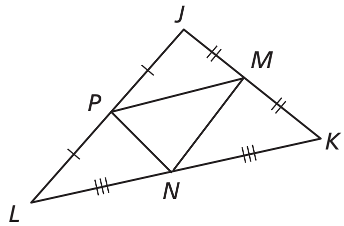

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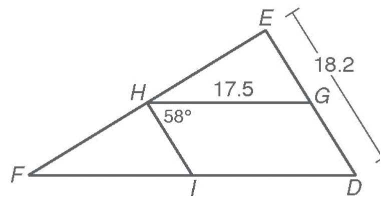
Additional Practice

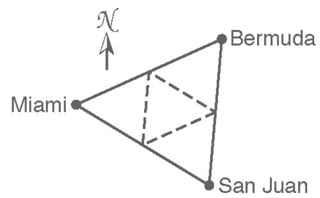


Use the figure for Exercises 1–6. Find each measure.

1. *HI* ____________________
2. *DF* ____________________
3. *GE* ____________________
4. m∠*HIF* ____________________
5. m∠*HGD* ____________________
6. m∠*D* ____________________

The Bermuda Triangle is a region in the Atlantic Ocean off the southeast coast of the United States. The triangle is bounded by Miami, Florida; San Juan, Puerto Rico; and Bermuda. In the figure, the dotted lines are midsegments.

	Dist. (mi)
Miami to San Juan	1038
Miami to Bermuda	1042
Bermuda to San Juan	965



7. Use the distances in the chart to find the perimeter of the Bermuda Triangle. ____________________
8. Find the perimeter of the midsegment triangle within the Bermuda Triangle. ____________________
9. How does the perimeter of the midsegment triangle compare to the perimeter of the Bermuda Triangle?

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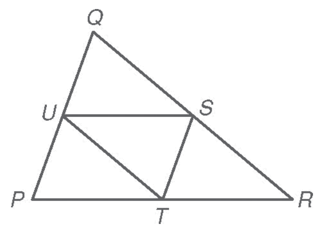
Write a two-column proof that the perimeter of a midsegment triangle is half the perimeter of the triangle.



10. **Given:** $\overline{US}$, $\overline{ST}$, and $\overline{TU}$ are midsegments of $\triangle PQR$.

Prove: The perimeter of $\triangle STU = \frac{1}{2}(PQ + QR + RP)$.

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Problem Solving

1. The vertices of $\triangle JKL$ are $J(-9, 2)$, $K(10, 1)$, and $L(5, 6)$. $\overline{CD}$ is the midsegment parallel to $\overline{JK}$. What is the length of $\overline{CD}$? Round to the nearest tenth.

____

2. In $\triangle QRS$, $QR = 2x + 5$, $RS = 3x - 1$, and $SQ = 5x$. What is the perimeter of the midsegment triangle of $\triangle QRS$?

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3. Is XY a midsegment of $\triangle LMN$ if its endpoints are $X(8, 2.5)$ and $Y(6.5, -2)$? Explain.

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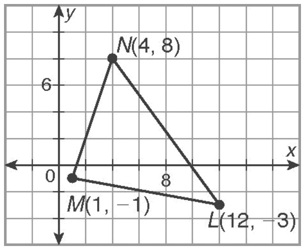

4. The diagram at right shows horseback riding trails. Point B is the halfway point along path $\overline{AC}$. Point D is the halfway point along path $\overline{CE}$. The paths along $\overline{BD}$ and $\overline{AE}$ are parallel. If riders travel from A to B to D to E, and then back to A, how far do they travel?

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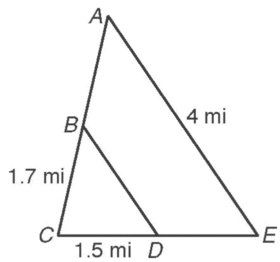


Choose the best answer.

5. Right triangle FGH has midsegments of length 10 centimeters, 24 centimeters, and 26 centimeters. What is the area of $\triangle FGH$?

   A 60 cm^2 C 240 cm^2

   B 120 cm^2 D 480 cm^2

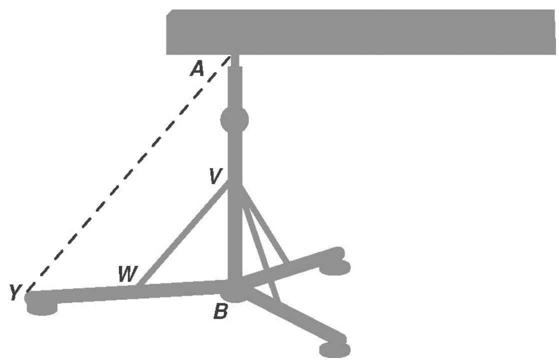
6. In triangle HJK, $m\angle H = 110°$, $m\angle J = 30°$, and $m\angle K = 40°$. If R is the midpoint of $\overline{JK}$, and S is the midpoint of $\overline{HK}$, what is $m\angle JRS$?

   F 150°   H 110°

   G 140°   J 30°

Use the diagram for Exercises 7 and 8.

On the balance beam, V is the midpoint of $\overline{AB}$, and W is the midpoint of $\overline{YB}$.



7. The length of $\overline{VW}$ is $1\frac{7}{8}$ feet. What is AY?

   A $\frac{7}{8}$ ft   C $3\frac{3}{4}$ ft

   B $\frac{15}{16}$ ft   D $7\frac{1}{2}$ ft

8. The measure of $\angle AYW$ is 50°. What is the measure of $\angle VWB$?

   F 45°   H 90°

   G 50°   J 130°

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