Geometry Game

2.6 Completing Proofs

MATERIALS

• game cards
• paper and pencil

HOW TO PLAY

• The game is played in groups of 2 to 4.
• Each player should have one game card.
• Separate the Angle, Segment and Equality Cards into three shuffled piles, with cards face down.
• On your turn, take a property card off the top of a pile. If it gives the property used in the next step of your proof, place it on that line. If not, discard it, face down. Keep the three types of cards separate.
• The winner is the first person to correctly match a property to each step of his or her proof.
• If you run out of property cards, shuffle the discard piles and use them for the draw piles. Not all property cards will be used.
Game Cards

CARD A

Given:
\[ m\angle 1 + m\angle 2 = 180^\circ \]
\[ m\angle 1 = 135^\circ \]

Prove:
\[ m\angle 2 = 45^\circ \]
\[ \frac{m\angle 1}{3} = m\angle 2 \]

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( m\angle 1 + m\angle 2 = 180^\circ )</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( m\angle 1 = 135^\circ )</td>
<td>2. Given</td>
</tr>
<tr>
<td>3. ( 135^\circ + m\angle 2 = 180^\circ )</td>
<td>3.</td>
</tr>
<tr>
<td>4. ( m\angle 2 = 45^\circ )</td>
<td>4.</td>
</tr>
<tr>
<td>5. ( \frac{m\angle 1}{3} = m\angle 2 )</td>
<td>5.</td>
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</tbody>
</table>

CARD B

Given:
\[ AC = 6 \text{ and } AB = 12 \]

Prove:
\[ CB = 6 \text{ and } C \text{ is the midpoint of } \overline{AB}. \]

<table>
<thead>
<tr>
<th>Statements</th>
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<tbody>
<tr>
<td>1. ( AC = 6 )</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( AB = 12 )</td>
<td>2. Given</td>
</tr>
<tr>
<td>3. ( AC + CB = AB )</td>
<td>3.</td>
</tr>
<tr>
<td>4. ( 6 + CB = 12 )</td>
<td>4.</td>
</tr>
<tr>
<td>5. ( CB = 6 )</td>
<td>5.</td>
</tr>
<tr>
<td>6. ( C \text{ is the midpoint of } \overline{AB}. )</td>
<td>6.</td>
</tr>
</tbody>
</table>
CARD C

Given:
\( m\angle 1 + m\angle 2 = 70^\circ \)
\( m\angle 1 = 35^\circ \)

Prove:
\( m\angle 2 = 35^\circ \)
\( \overrightarrow{BD} \) is the angle bisector of \( \angle ABC \)

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
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<tbody>
<tr>
<td>1. ( m\angle 1 + m\angle 2 = 70^\circ )</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( m\angle 1 = 35^\circ )</td>
<td>2. Given</td>
</tr>
<tr>
<td>3. ( 35^\circ + m\angle 2 = 70^\circ )</td>
<td>3.</td>
</tr>
<tr>
<td>4. ( m\angle 2 = 35^\circ )</td>
<td>4.</td>
</tr>
<tr>
<td>5. ( \angle 1 \cong \angle 2 )</td>
<td>5.</td>
</tr>
<tr>
<td>6. ( BD ) is the angle bisector of ( \angle ABC )</td>
<td>6.</td>
</tr>
</tbody>
</table>

CARD D

Given:
\( CB = 9 \) and \( AB = 12 \)

Prove:
\( AC = 3 \) and \( AC = \frac{1}{4} AB \)

<table>
<thead>
<tr>
<th>Statements</th>
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<tbody>
<tr>
<td>1. ( CB = 9 )</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( AB = 12 )</td>
<td>2. Given</td>
</tr>
<tr>
<td>3. ( AC + CB = AB )</td>
<td>3.</td>
</tr>
<tr>
<td>4. ( AC + 9 = 12 )</td>
<td>4.</td>
</tr>
<tr>
<td>5. ( AC = 3 )</td>
<td>5.</td>
</tr>
<tr>
<td>6. ( AC = \frac{1}{4} AB )</td>
<td>6.</td>
</tr>
<tr>
<td>Property Cards</td>
<td></td>
</tr>
<tr>
<td>----------------</td>
<td></td>
</tr>
<tr>
<td><strong>Segment Addition Postulate</strong></td>
<td><strong>Angle Addition Postulate</strong></td>
</tr>
<tr>
<td>Definition of congruent segments</td>
<td>Definition of congruent angles</td>
</tr>
<tr>
<td>Definition of midpoint</td>
<td>Definition of angle bisector</td>
</tr>
<tr>
<td>Substitution Property of Equality</td>
<td>Substitution Property of Equality</td>
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<tr>
<td>Substitution Property of Equality</td>
<td>Substitution Property of Equality</td>
</tr>
<tr>
<td>Subtraction Property of Equality</td>
<td>Subtraction Property of Equality</td>
</tr>
<tr>
<td>Division Property of Equality</td>
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</tr>
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<td>Subtraction Property of Equality</td>
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</table>
# Answer Key

## CARD A

**Given:**
\[ m\angle 1 + m\angle 2 = 180^\circ \]
\[ m\angle 1 = 135^\circ \]

**Prove:**
\[ m\angle 2 = 45^\circ \]
\[ \frac{m\angle 1}{3} = m\angle 2 \]

**Statements**
1. \( m\angle 1 + m\angle 2 = 180^\circ \)
2. \( m\angle 1 = 135^\circ \)
3. \( 135^\circ + m\angle 2 = 180^\circ \)
4. \( m\angle 2 = 45^\circ \)
5. \( \frac{m\angle 1}{3} = m\angle 2 \)

**Reasons**
1. Given
2. Given
3. Substitution Property of Equality
4. Subtraction Property of Equality
5. Division Property of Equality

## CARD B

**Given:**
\( AC = 6 \) and \( AB = 12 \)

**Prove:**
\( CB = 6 \) and \( C \) is the midpoint of \( \overline{AB} \).

**Statements**
1. \( AC = 6 \)
2. \( AB = 12 \)
3. \( AC + CB = AB \)
4. \( 6 + CB = 12 \)
5. \( CB = 6 \)
6. \( C \) is the midpoint of \( \overline{AB} \).

**Reasons**
1. Given
2. Given
3. Segment Addition Postulate
4. Substitution Property of Equality
5. Subtraction Property of Equality
6. Definition of midpoint
CARD C

Given:
\( m\angle 1 + m\angle 2 = 70^\circ \)
\( m\angle 1 = 35^\circ \)

Prove:
\( m\angle 2 = 35^\circ \)
\( \overrightarrow{BD} \) is the angle bisector of \( \angle ABC \)

Statements
1. \( m\angle 1 + m\angle 2 = 70^\circ \)
2. \( m\angle 1 = 35^\circ \)
3. \( 35^\circ + m\angle 2 = 70^\circ \)
4. \( m\angle 2 = 35^\circ \)
5. \( \angle 1 \cong \angle 2 \)
6. \( \overrightarrow{BD} \) is the angle bisector of \( \angle ABC \)

Reasons
1. Given
2. Given
3. Substitution Property of Equality
4. Subtraction Property of Equality
5. Definition of Congruent Angles
6. Definition of Angle Bisectors

CARD D

Given:
\( CB = 9 \) and \( AB = 12 \)

Prove:
\( AC = 3 \) and \( AC = \frac{1}{4} AB \)

Statements
1. \( CB = 9 \)
2. \( AB = 12 \)
3. \( AC + CB = AB \)
4. \( AC + 9 = 12 \)
5. \( AC = 3 \)
6. \( AC = \frac{1}{4} AB \)

Reasons
1. Given
2. Given
3. Segment Addition Postulate
4. Substitution Property of Equality
5. Subtraction Property of Equality
6. Division Property of Equality
Teacher Notes

ACTIVITY PREPARATION

- Photocopy the Game Cards and the Property Cards. You may want to use a different color for the different Property Cards. If you plan to re-use the game pieces, use colored card stock or heavier weight paper, and laminate. Cut the Property Cards into strips. Store the game pieces in an envelope or sandwich bag. Make enough sets so that students can play in groups of 2 to 4.

ACTIVITY MANAGEMENT

- The cards are laid face down on the table in a grid. Recommend to students that they separate the two types of cards. Player One selects a card of each type. If there is a match then Player One keeps the cards and takes another turn. If they do not match the cards are turned faced down again and the next player picks two cards. Play continues until all cards have been matched. If you wish to name a winner, the player with the most matches is named the winner.
- **A-Level Alternative** Have students pair the cards as a puzzle rather than as a game. Place all cards face up and create pairs.
- **C-Level Alternative** Have students pair mix all of the property cards together. Have students complete two game cards at the same time.
Activity and Closure Questions

Answer these questions as a class.

1. What is the difference between congruence and equality?
   
   Answer: “Congruent” describes two different figures that have the same shape and size. Their corresponding measures are equal. “Equality” refers to quantities that are the same, or to expressions that have the same value.

2. What is a postulate?
   
   Answer: A postulate is an idea that is universally accepted without further proof. We cannot base a postulate upon any other idea. It is a building block for other ideas that we can use.

LESSON TRANSITION

Students should play this game after Lesson 2.6. Writing a proof is one of the most novel and most difficult skills that students learn in geometry, and this activity is intended to provide informal practice with writing simple proofs before students write more complicated proofs in subsequent lessons.